



**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**

**B.Sc. DEGREE EXAMINATION – MATHEMATICS**

**FIRST SEMESTER – APRIL 2023**

**UMT 1502 – CALCULUS**

Date: 09-05-2023

Dept. No.

Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

**PART – A**

**Answer all the questions:**

**(10 × 2 = 20)**

1. State Leibnitz formula for the derivative of the product of two functions.
2. Find  $n^{th}$  derivative of  $\frac{1}{ax+b}$ .
3. Find the slope of the tangent with the initial line for the cardioid  $r = a(1 - \cos\theta)$  at  $\theta = \frac{\pi}{6}$ .
4. Write the formula to find radius of curvature of  $y = f(x)$ .
5. Evaluate  $\int (2x + 3)^2 dx$ .
6. State any two properties of definite integral.
7. Evaluate  $\int_0^1 \int_0^1 (x + y) dx dy$ .
8. Define Jacobian of two variables u,v with respect to x,y.
9. Define gamma function.
10. State any two properties of beta function.

**PART B**

**Answer any Five of the following:**

**(5 × 8 = 40)**

11. Find the coordinates of centre of curvature of  $xy = 2at$  (2,1).
12. If  $y = a \cos(\log x) + b \sin(\log x)$ , prove that  $x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$ .
13. Find the angle of intersection of cardioids  $r = a(1 + \cos\theta)$  and  $r = b(1 - \cos\theta)$
14. Evaluate  $\int_0^1 x^m \log\left(\frac{1}{x}\right)^n dx$ .
15. Find a reduction formula for  $\int \sin^n x dx$ , where n is a positive integer.
16. By changing the order of integration, evaluate  $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dx dy$ .
17. Prove that  $\Gamma(n+1) = n!$
18. Evaluate (i)  $\int_0^1 x^7 (1 - x)^8 dx$  \_\_\_\_\_ (4 marks)  
(ii)  $\int_0^{\frac{\pi}{2}} \sin^{10} \theta d\theta$  \_\_\_\_\_ (4 marks)

**PART – C**

**Answer any Two of the following:**

**(2 × 20 = 40)**

19. Show that the maximum value of  $x^2 y^2 z^2$  subject to the restriction  $x^2 + y^2 + z^2 = a^2$  is  $\left(\frac{a^2}{3}\right)^3$ .
20. Find the value of the integral  $\iiint xyz dx dy dz$  taken through the positive octant of the sphere  $x^2 + y^2 + z^2 = a^2$ .
21. Find the evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .
22. Derive the relationship between beta and gamma function.

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